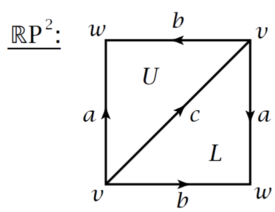
MAT 762, Algebraic Topology, Fall 2013

Homework Assignment 2

Problem 1. Given two abelian groups M and N, one can define $\mathrm{Ext}(M,N)$ and $\mathrm{Tor}(M,N)$ by $\mathrm{Ext}(M,N) := H^1(\mathrm{Hom}(C_*,N))$ and $\mathrm{Tor}(M,N) := H_1(C_* \otimes N)$ where $C_* \to M \to 0$ is a free resolution of M. Compute $\mathrm{Hom}(M,N)$, $M \otimes N$, $\mathrm{Ext}(M,N)$, and $\mathrm{Tor}(M,N)$ for all $(M,N) \in \{\mathbb{Z}, \mathbb{Z}_m\} \times \{\mathbb{Z}, \mathbb{Z}_n\}$.

Problem 2. Compute the simplicial cohomology of the following Δ-complex in two different ways: (a) directly from the definitions, and (b) by using the Universal Coefficient Theorem and Problem 3 of the previous homework assignment.

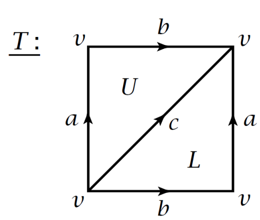


Problem 3. (Hatcher, Section 3.1, Exercise 5) Regarding a cochain $\alpha \in C^1(X)$ as a function from paths in X to $\mathbb{Z}$, show that if α is a cocycle, then

(a) $\alpha(f \cdot g) = \alpha(f) + \alpha(g)$,
(b) α takes the value 0 on constant paths,
(c) $\alpha(f) = \alpha(g)$ if f is homotopic to g with fixed endpoints,
(d) α is a coboundary iff $\alpha(f)$ depends only on the endpoints of f, for all f.

Remarks. (a) and (c) give a map $H^1(X) \to \mathrm{Hom}(\pi_1(X), \mathbb{Z})$, which the Universal Coefficient Theorem says is an isomorphism if X is path-connected. The proof of (b) and (c) would be somewhat more obvious if α were a cubical singular 1-cocycle.

Problem 4. For the following Δ-complex, compute the simplicial cup product $(a^* - b^*) \cup (b^* + c^*)$ and show that it is nonzero in the simplicial cohomology of $T \cong S^1 \times S^1$. Conclude that $H^*(T)$ and $H^*(S^1 \vee S^1 \vee S^2)$ are isomorphic as abelian groups but not as graded rings.



This homework is due on Tuesday, October 1, 2013.