Publications and Preprints

Please note that, in pure mathematics, authors on collaborative papers are listed in alphabetical order.

1. **On unification of colored annular $\mathfrak{sl}_2$ knot homology** (with A. Beliakova, M. Hogancamp, and K. Putyra), submitted, available at arXiv:2305.02977, 47 pages

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The main goal of this paper is to compare two different models of non-reduced colored $\mathfrak{sl}_2$ knot homology: Khovanov's model from 2003, and Cooper–Krushkal's model from 2010. For the zero-framed annular unknot, we prove that these two models become homotopy equivalent when formulated in the quantum annular Bar-Natan category. Again for the unknot, we show that these two models are equivalent to a third one, which is defined using an action of the Jones–Wenzl projectors on the quantum annular homology of cables.

We further give a new construction of the Cooper–Hogancamp projectors P_ε, which are certain categorical idempotents that categorify projectors onto irreducible summands in tensor powers of the fundamental representation of quantum $\mathfrak{sl}_2$.

For k the number of through-strands in P_ε, we prove that the class of P_ε in the quantum horizontal trace agrees with that of the Cooper–Krushkal projector P_k, provided q is generic. As an application, we compute the full quantum Hochschild homology of Khovanov's arc ring H^n. Finally, we state precise conjectures formalizing cabling operations and extending the above results to all knots.

2. **On the functoriality of $sl(2)$ tangle homology** (with A. Beliakova, M. Hogancamp, and K. Putyra), Algebraic & Geometric Topology, **23** (2023) 1303–1361

One of the important features of Khovanov homology is that it extends to a functor which assigns a linear map ϕ_S to each smooth link cobordism $S \subset \mathbb{R}^3 \times I$. While the map ϕ_S is an invariant of the smooth isotopy class of S, it is a priori only well-defined up to an overall sign. Various approaches for fixing the sign in ϕ_S have been proposed. Two of these approaches, found independently by Blanchet and Caprau, are based on using $\mathfrak{gl}_2$ foams instead of the more traditional merge and split cobordisms that appear in the original definition of Khovanov homology.

In our paper, we aim to extend the known functoriality results to tangles (i.e., to parts of knots and links). To this end, we establish an equivalence between a weighted Bar-Natan bicategory $\mathbf{wBN} = \mathbf{BN} \times \mathbb{Z}$ and a bicategory **Foam** of $\mathfrak{gl}_2$ foams. Using this equivalence, we then conclude that certain tangle invariants derived from Khovanov homology are properly functorial (not just up to sign!) with respect to smooth tangle cobordisms.

In the final sections of our paper, we further introduce web versions of Khovanov's arc algebras H^n and of the Chen–Khovanov algebras A^n. An immediate application is a strictly functorial version of the "quantized" annular Khovanov homology defined in my paper 5 with Beliakova and Putyra.

One of the main observation in our paper is that each $\mathfrak{gl}_2$ foam embedded in a 3-ball corresponds to a pair of transversely embedded surfaces. Using this observation, we can reinterpret relations among foams geometrically, in terms of isotopies of surfaces paired with sign changes.

3. **Odd annular Bar-Natan category and $\mathfrak{gl}(1|1)$** (with C. Necheles), submitted, available at arXiv:2206.01892, 83 pages

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In this joint paper with my former graduate student Casey Necheles, we provide a conceptual interpretation of the $\mathfrak{gl}(1|1)$-action described in my paper 4. To this end, we introduce two monoidal supercategories: the odd dotted Temperley–Lieb supercategory $\mathcal{TL}_{o,\bullet}(\delta)$, and the odd annular Bar-Natan category $\mathcal{BN}_o(\mathbb{A})$, which generalizes Putyra's non-annular odd cobordism category.

Next, we prove in two different ways that $\mathcal{TL}_{o,\bullet}(0)$ embeds fully faithfully into $\mathcal{BN}_o(\mathbb{A})$. Our first proof uses Reeb graphs of chronological cobordisms and provides an explicit left-inverse for the embedding. In contrast, the second proof is more conceptual and uses the connection with the non-annular odd Bar-Natan category. Using the second proof, we also obtain explicit bases for the morphism sets of $\mathcal{BN}_o(\mathbb{A})$.

By relating $\mathcal{TL}_{o,\bullet}(0)$ to the representation category of $\mathfrak{gl}(1|1)$, we then show that odd annular Khovanov homology carries a natural $\mathfrak{gl}(1|1)$-action. Finally, we prove that this $\mathfrak{gl}(1|1)$-action is isomorphic to the one from my earlier paper 4.

4. **An action of $\mathfrak{gl}(1|1)$ on odd annular Khovanov homology** (with J. E. Grigsby), *Mathematical Research Letters*, **27**(3) (2020) 711–742

In this paper, we generalize some of the results from our publication 7 to the odd setting. More concretely, we define an annular version of odd Khovanov homology, and we prove that it carries an action not of the Lie algebra $\mathfrak{sl}_2$, but of the Lie superalgebra $\mathfrak{gl}(1|1)$. This $\mathfrak{gl}(1|1)$-action is preserved under maps induced by Reidemeister moves and can be defined in two different ways: either by using the original definition of odd Khovanov homology, or by adapting the construction of the $\mathfrak{sl}_2$-action that exists in the even setting. A more intrinsic construction of the $\mathfrak{gl}(1|1)$-action on odd annular Khovanov homology is given in my recent paper 3.

5. **Quantum Link Homology via Trace Functor I** (with A. Beliakova and K. Putyra), *Inventiones Mathematicae*, 215(2) (2019) 383–492

In this 110 page article, we prove a conjecture relating annular Khovanov homology to the Chen–Khovanov tangle invariant, and we develop a general theory of twisted categorified trace functions (or shadows) on a bicategory $\mathbf{C}$ equipped with an endofunctor $\Sigma\colon \mathbf{C} \to \mathbf{C}$. As main examples of categorified trace functions, we consider Hochschild homology and twisted Hochschild–Mitchell homology. To a pair $(\mathbf{C}, \Sigma)$ as above, we further assign category $\mathrm{hTr}(\mathbf{C}, \Sigma)$, which generalizes the horizontal trace defined by Beliakova–Habiro–Lauda–Živković, and which under some assumption forms the target category of a universal Σ-twisted shadow on $\mathbf{C}$.

For a graded linear bicategory and a fixed invertible parameter q, we then "quantize" our constructions by replacing Σ by an endofunctor Σ_q such that $\Sigma_q\alpha := q^{-|\alpha|}\Sigma\alpha$ for all homogeneous 2-morphisms α. This leads to the notion of a quantum horizontal trace, and we show that the quantum horizontal trace of the Bar-Natan bicategory $\mathbf{BN}$ can be described geometrically as the quantum annular Bar-Natan category $\mathcal{BN}_q(\mathbb{A})$.

On the algebra side, we use ideas of Keller to prove a general result about the quantum Hochschild homology of certain finite-dimensional graded algebras of finite global dimension.

By combining our ideas, we then show that the Hochschild homology of the Chen-Khovanov tangle invariant agrees with the annular Khovanov homology of the tangle closure. This was previously conjectured in my paper 10, where it was partially shown in a special case using different methods. By replacing Hochschild homology by quantum Hochschild homology, we finally obtain a new triply-graded homology theory for knots and links contained in a thickened annulus.

This "quantized" annular Khovanov homology carries a natural action of the quantum group $\mathcal{U}_q(\mathfrak{sl}_2)$, and it gives rise to nontrivial invariants for closed oriented surfaces embedded in a 4-dimensional solid torus. In the case where the embedded surface is obtained by taking the closure of a link cobordism $W\colon L \to L$, for a link $L \subset \mathbb{R}^3$, the associated invariant turns out to be the graded Lefschetz number of the action of W on the Khovanov homology of L. In particular, we obtain an interpretation of the Jones polynomial as an invariant assigned to the surface $L \times S^1 \subset \mathbb{R}^3 \times S^1$.

6. **Annular Khovanov-Lee homology, braids, and cobordisms** (with J. E. Grigsby and A. Licata), *Pure and Applied Mathematics Quarterly, Special Issue in Honor of Simon Donaldson*, **13**(3) (2018) 389–436

In this paper, we prove that the Khovanov–Lee complex of an oriented link L in a thickened annulus, $\mathbb{A} \times I$, has the structure of a $(\mathbb{Z} \oplus \mathbb{Z})$-filtered chain complex whose filtered homotopy type is an invariant of the isotopy class of $L \subset \mathbb{A} \times I$.

Using ideas of Ozsváth–Stipsicz–Szabó, we use this structure to define a family of annular Rasmussen invariants that yield information about annular and non-annular link cobordisms. Our family of invariants gives rise to a piecewise linear function on the interval $[0, 2]$ which is symmetric along $t = 1$. Focusing on the special case of annular links obtained as braid closures, we use the behavior of this function to obtain a necessary condition for braid quasipositivity and a sufficient condition for right-veeringness.

7. **Annular Khovanov homology and knotted Schur-Weyl representations** (with J. E. Grigsby and A. Licata), *Compositio Mathematicae*, **154**(3) (2017) 459–502

In this article, we show that the annular Khovanov homology of a link L in a thickened annulus $\mathbb{A} \times I$ carries a natural action of the Lie algebra $\mathfrak{sl}_2$. More generally, we show that this action extends to an action of a certain Lie superalgebra, which we refer to as the exterior current algebra of $\mathfrak{sl}_2$.

In the case where the link L is an n-cable of a framed knot $K \subset \mathbb{A} \times I$, we show that there is a commuting action of the symmetric group $\mathfrak{S}_n$, which is defined in terms of cobordims maps. We therefore obtain a "knotted Schur–Weyl representation" which agrees with classical $\mathfrak{sl}_2$ Schur–Weyl duality when the annular knot K is the zero-framed essential unknot.

Some of the results of this paper have since been generalized by Queffelec–Rose and in my recent papers 2, 4, 5. Moreover, I am currently working on a joint project with my graduate student Jacob Migdail-Smith, which aims to show among other things that the odd Khovanov homology of an n-cable carries an action not of the symmetric group $\mathfrak{S}_n$, but of a Hecke algebra at $q = -1$.

8. **An elementary fact about unlinked braid closures** (with J. E. Grigsby), *AWM Springer Series, Advances in the Mathematical Sciences,* (2016) 93–101

In this article, we use Khovanov homology to prove the following fact: if the closure of an n-strand braid is the n-component unlink, then the braid is the n-strand identity braid. We also use Heegaard Floer homology to prove a related result about fibered links in connected sums of $S^1 \times S^2$'s. While both of these results were previously known to some experts (they can be proven by using the fundamental group), the main goal of our paper is to illustrate how Khovanov homology and Heegaard Floer homology can be used to study certain questions related to braids and fibered links.

9. **A note on the combinatorics of lens space d-invariants** (with M. Doig), submitted, available at arXiv:1505.06970, 10 pages

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In this paper, we use the combinatorics of the Heegaard Floer d-invariants to study the homology cobordism classification of 3-dimensional lens spaces. Specifically, we use a formula of Lee–Lipshitz for the lens space d-invariants to to establish that two 3-dimensional lens spaces are integer homology cobordant exactly when they are oriented homeomorphic. This result also follows implicitly from recent work in Heegaard Floer theory (and it was previously shown in special cases by Gilmer–Livingston and Fintushel–Stern), but ours is the first combinatorial proof, modulo the fact that the d-invariants are well-defined. In particular, our proof bypasses the connection with Turaev torsion.

In our paper, we also formulate a conjecture about the homology cobordism classification of L-spaces surgeries on the trefoil. Note that the arXiv version of this paper does not contain this conjecture and appeared under a different title.

10. **Sutured Khovanov homology, Hochschild homology, and the Ozsváth-Szabó spectral sequence** (with D. Auroux and J. E. Grigsby), *Transactions of the American Mathematical Society* **367**(10) (2015) 7103–7131

In 2001, Khovanov and Seidel constructed a faithful action of the $(m+1)$-strand braid group on the derived category of left modules over a certain quiver algebra A_m. Explicitly, this action is given by assigning to each braid β on $m+1$ strands a complex of bimodules over A_m. In this paper, we interpret the Hochschild homology of this complex as a direct summand of the annular Khovanov homology of the annular closure of β. We further formulate a conjecture which relates the full annular Khovanov homology of the annular closure of β to the Hochschild homology of the Chen–Khovanov invariant of β. This conjecture was proven in full generality in my recent paper 5.

11. **Khovanov-Seidel quiver algebras and bordered Floer homology** (with D. Auroux and J. E. Grigsby), *Selecta Mathematica* **20**(1) (2014) 1–55

Like my paper 10, this article deals with the braid action of Khovanov–Seidel. We start by giving an explicit description of the Ext algebra $B_m = \mathrm{Ext}(V_m, V_m)$, where V_m denotes the direct sum of all standard modules of the Khovanov–Seidel algebra A_m. Given a braid β in the solid cylinder $D^2 \times I$, we then define a filtered $\mathcal{A}_\infty$-bimodule with the following two properties: (1) the associated graded $\mathcal{A}_\infty$-bimodule is Morita equivalent to the Khovanov–Seidel invariant of β; and (2) the underlying unfiltered $\mathcal{A}_\infty$-bimodule agrees with the bordered Floer bimodule associated to the branched double-cover of $D^2 \times I$, branched along β.

This result is significant in so far as it provides a local version of (a part of) the previously known relationship between Khovanov homology and Heegaard Floer homology.

Moreover, the derived category of B_m can be interpreted conjecturally as a Yoneda embeddings of a partially wrapped Fukaya category of a Lefschetz fibration. Similarly, the relevant category on the bordered Floer side can be seen as a Yoneda embedding of a partially wrapped Fukaya category of a Riemann surface. At least conjecturally, our result thus provides a relationship between two monodromy action of the braid group on two different Fukaya categories.

12. **On Gradings in Khovanov homology and sutured Floer homology** (with J. E. Grigsby), *Contemporary Mathematics* **560** (Proceedings of William Jaco's 70th birthday conference) (2011) 111–128

The aim of this paper is to provide an easily digestible account, complete with representation-theoretic and geometric background, of a relationship between certain gradings that appear in annular Khovanov homology and in sutured Heegaard Floer homology. More specifically, we show that these gradings correspond to each other under the spectral sequence from our paper 14.

13. **On the naturality of the spectral sequence from Khovanov homology to Heegaard Floer homology** (with J. E. Grigsby), *International Mathematics Research Notices* **2010**(21) (2010) 4159–4210

In our papers 14 and 16, we observed that the Ozsváth-Szabó spectral sequence between Khovanov homology and Heegaard Floer homology admits a generalization involving sutured Floer homology (a variant of Heegaard Floer homology due to Juhász). In the present paper, we show that this generalized spectral sequence behaves nicely under decomposition of sutured 3-manifolds along certain embedded surfaces. As an application, we deduce two results related to stacking of tangles.

14. **Khovanov homology, sutured Floer homology, and annular links** (with J. E. Grigsby), *Algebraic & Geometric Topology* **10** (2010) 2009–2039

Around 2007, L. Roberts defined an Ozsváth–Szabó-type spectral sequence between the annular Khovanov homology of an annular link $L \subset \mathbb{A} \times I$ and the knot Floer homology of a knot related to the link L. Using this spectral sequence, he was able to establish a relationship between Plamenevskaya's invariant for transverse links and Ozsváth–Szabó's invariant for contact 3-manifolds. In the present paper, we use sutured Floer homology to give a reinterpretation of Roberts's spectral sequence. In addition, we show that the spectral sequence from our paper 16 is a direct summand of Roberts's spectral sequence.

15. **Mutation invariance of Khovanov homology over $\mathbb{F}_2$**, *Quantum Topology* **1**(2) (2010) 111–128

Like my paper 21, this paper deals with the behavior of Khovanov homology under a certain cut-and-paste operation known as Conway mutation. While my paper 21 provided examples of mutant links with different integer Khovanov homology, the present paper shows that Khovanov homology with coefficients in $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$ is invariant under some link mutations including all knot mutations. The proof of this result uses a differential that can be defined on the morphism sets of a Bar-Natan category over $\mathbb{F}_2$, as well as an idea that was first proposed by Bar-Natan.

A very different proof of a related result was found independently by Bloom. Very recently, Kotelskiy–Watson–Zibrowius established an analogous result for (even) Khovanov homology over an arbitrary field.

16. **On the Colored Jones Polynomial, Sutured Floer homology, and Knot Floer homology** (with J. E. Grigsby), *Advances in Mathematics* **223**(6) (2010) 2114–2165

Building on the work of Ozsváth–Szabó, we use pseudo-holomorphic polygons to establish an explicit relationship, in the form of a spectral sequence, between the sutured Khovanov homology of a balanced tangle T in the solid cylinder $D^2 \times I$ and the sutured Floer homology of the branched double-cover of $D^2 \times I$, branched along the mirror image of T. Restricting to a special case, we obtain a spectral sequence between Khovanov's categorification of the reduced colored Jones polynomial and a variant of knot Floer homology. We then use this spectral sequence to prove that Khovanov's categorificaion of the reduced n-colored Jones polynomial detects the unknot for all $n > 1$.

At the time of its discovery, this was one of the best existing result in the direction of a still open conjecture, which asserts that the Jones polynomial detects the unknot. More recently, Kronheimer–Mrowka and Dowlin have used different spectral sequences to prove that ordinary Khovanov homology (corresponding to the $n = 1$ case) detects the unknot.

In our paper, we also formulate a conjecture about genus bounds derived from gradings on reduced colored Khovanov homology.

17. **A remark on the topology of (n, n) Springer varieties**, available at arXiv:0908.2185

Springer varieties play appear in geometric representation theory because their cohomology rings carry an action of the symmetric group. In 2002, Khovanov proved that the cohomology of the (n, n) Springer variety $\mathcal{B}_{n,n}$ is isomorphic to the center of his arc ring H^n. In this context, he introduced a space $\widetilde{S}$, which is built from products of 2-spheres, and which has the same cohomology ring as $\mathcal{B}_{n,n}$. In the present paper, I use ideas of Cautis–Kamnitzer to show that this space is, in fact, homeomorphic to $\mathcal{B}_{n,n}$. The same result was obtained independently by Russell–Tymoczko.

18. **Categorification of the colored Jones polynomial and Rasmussen invariant of links** (with A. Beliakova), *Canadian Journal of Mathematics* **60** (2008) 1240–1266

This paper is divided into two largely separate parts. In the first part, we introduce a two-parameter family of deformations of Khovanov's categorification of the non-reduced n-colored Jones polynomial. We define these deformed colored Khovanov complexes over $\mathbb{Z}[1/2]$ coefficients and in Bar-Natan's geometric framework. We then study conditions under which framed colored link cobordisms induce chain maps between our colored Khovanov brackets. In this context, we also develop a notion of movies and movie moves for framed link cobordisms (generalizing the unframed movie moves of Carter–Saito).

In a second part of the paper, we generalize Rasmussen's s-invariant from knots to links, and we give examples where this generalized Rasmussen invariant is a stronger obstruction to sliceness than the multivariable Levine–Tristram signature.

19. **A spanning tree model for Khovanov homology**, *Journal of Knot Theory and Its Ramifications* **17**(12) (2008) 1561–1574

In this paper, I prove that Khovanov's chain complex deformation retracts to a usually much smaller chain complex whose generators are in 2:1 correspondence with the spanning trees of the checkerboard graph of the knot diagram. Using a graded version of this result, I also give a new proof of a theorem of E. S. Lee about the support of Khovanov's invariants for alternating knots. Moreover, I give a short proof (unrelated to spanning trees) of Lee's theorem about the degeneration of Lee homology.

A spanning tree model for Khovanov homology was also found independently by Champanerkar–Kofman. More recently, somewhat different spanning tree constructions were studied by various authors, including Roberts and Jaeger.

20. **Contributions to Khovanov homology**, Ph.D. Thesis, University of Zurich (2007), 81 pages

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My thesis contains the results from my papers 18, 19, and 21. In addition, my thesis provides a self-contained introduction to Khovanov homology and to Bar-Natan's universal theory, as well as some background information on Frobenius systems.

21. **Khovanov Homology and Conway Mutation**, available at arXiv:math/0301312, 9 pages

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In this paper, I give a simple example of mutant links with different integer Khovanov homology. The existence of such an example is noteworthy because it shows that Khovanov homology cannot be computed via a skein rule similar to the skein relation for the Jones polynomial. More immediately, my example shows that links with the same Jones polynomial can have different Khovanov homology (but links with this property were known before, although there was no conceptual explanation for the existence of such links).